Modeling of plasticity in spectral methods for fatigue damage estimation of random vibrations.

H .Rognon[(a)], T.Da Silva Botelho[(a)], I.Tawfiq[(a)], A.Galtier[(b)],M.Bennebach[(b)]*

a. LISMMA (EA2336) Supméca, 3 rue Fernand Hainaut 93407 Saint-Ouen cedex, France

b. CETIM, 52 Avenue Félix Louat BP 80067, 60304 Senlis Cedex, France

Absract :

Spectral methods are used to estimate fatigue damage under random vibration via structural dynamic responses. By integrating the confined elasto-plastic behavior of materials in spectral methods we increase their range of use. These phenomena occur in the case of accelerated vibratory fatigue tests. Neuber method combined with the fatigue law of Manson-Coffin-Basquin in spectral methods allows taking into account the elastoplastic behavior of some materials. This correction is validated via a comparison with Monte-Carlo simulations under narrow band and broadband excitation.

Résumé :

Les méthodes spectrales sont des méthodes d'estimation du dommage par fatigue dans le cadre de vibrations aléatoires et d'une réponse dynamique de la structure. L'intégration du comportement élasto-plastique confiné des matériaux dans les méthodes spectrales permet d'augmenter leurs domaines d'utilisation. Ces phénomènes se produisent dans le cas d'essais de fatigue vibratoire accélérés. L'utilisation de la méthode de Neuber combinée avec la loi de fatigue Manson-Coffin-Basquin au sein des méthodes spectrales permettent la prise en compte du comportement élasto-plastique de certains matériaux. Cette correction est validée via une comparaison avec des simulations de Monte-Carlo sur des excitations à bande étroite et à bande large.

Introduction

The phenomenon of fatigue is now a major design criterion in the engineering industry. The fatigue design methods should best model real physical phenomena. Conventional methods of computing fatigue are defined in time domain because their input data is the stress time history. The algorithms such as Rainflow [1,2] extract stress cycles from the time history to obtain a stress distribution. To determine the lifetime generated by the stress distribution, we can used a fatigue law such as Basquin equation and accumulation law such as Palmgren-Miner law. Nevertheless, these methods require knowing stress time history or envelope of this time history. In the case of random vibrations, the mechanical structure has a dynamic response. It is difficult to know the stress history and to make the fatigue analysis because:

- Integral convolution increases computing time
- Random nature of process

To overcome these problems we try to study fatigue phenomenon in frequency domain. In addition, we model the random vibrations as a Gaussian stochastic process. In the spectral domain, the Gaussian stochastic process is characterized by its PSD (power spectral density).

Fatigue spectral methods are entirely in the frequency domain. The expected damage is then calculated from the PSD of stresses. The cycles distribution is not calculated but is estimated with a probability density of the maxima. This probability density depends on parameters of the DSP and the spectral methods selected.

Spectral methods have shown their reliability in the calculation fatigue. However, several assumptions are needed to apply these spectral methods. These Assumptions reduce the area of application. To increase the area of use of these spectral methods, we modeled the elasto-plastic behavior of materials in confined situations and we use a fatigue law that take into account the plasticity of materials.

General Purpose

Fatigue law

In classical spectral methods, the commonly fatigue model used is the Basquin equation [3]. This gives a power law relation between stress amplitude and life. This model leads to a linear Stress-Life curve in a log-log scale. It is defined in Eqn. 1:

$$N\sigma^{\beta} = C \qquad (1)$$

With β and C material constants

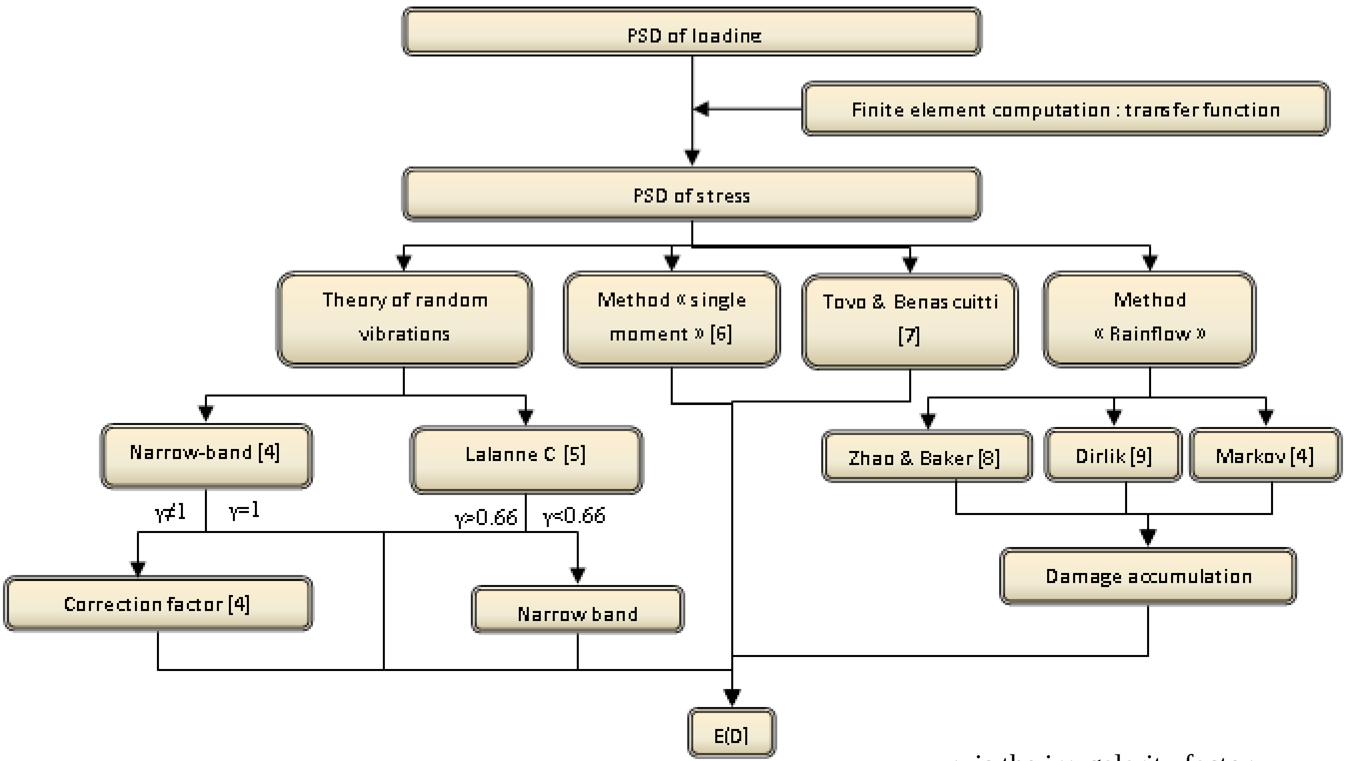
In the case of the proposed approach, the fatigue model used is Basquin-Manson-Coffin's [3]. This is a non linear model that determines total strain amplitude as a function of life. It is governed by Eqn.2:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma_f}{E}(2N)^b + \varepsilon_f (2N)^c \qquad (2)$$

Where ε_f , σ_f , b and c material's fatigue law constants.

Spectral methods

As said in the introduction, the cycle number distribution is not computed from a stress temporal representation. But the distribution is obtained from the probability density of maxima. The definition of the probability density of maxima differs according to the methods. Some are based on the theory of random vibrations, and others, on the definition of a Rainflow cycle. Figure 1 shows the spectral methods existing in the literature



γ is the irrugalarity factor

Figure 1 : Existing spectral methods

The spectral methods are subject to several assumptions such as linearity (material and/or geometric). These assumptions provide a correct computation of stress PSD and proper use of the fatigue law.

Theory

Formulation of spectral methods in strain

We have seen before that the spectral methods were formulated in stress. If the mechanical structure works only with an elastic behavior, the choice of work in stress formulation is largely justified. In stress, there is linear fatigue laws that give good damage estimation. In addition, the stress formulations can be explicit and are used quickly and easily without special means. For against, if mechanical structure has a confined elasto-plastic behavior, it requires fatigue law that takes into account the plasticity of the material. We choose to work with Coffin-Manson-Basquin model (Eqn. 2) as we can know the elastic and plastic contributions parts of strain. This assumes that the function ε = f (N) is an invertible function. This change in fatigue law involves rewriting the spectral methods in strain (fig2).

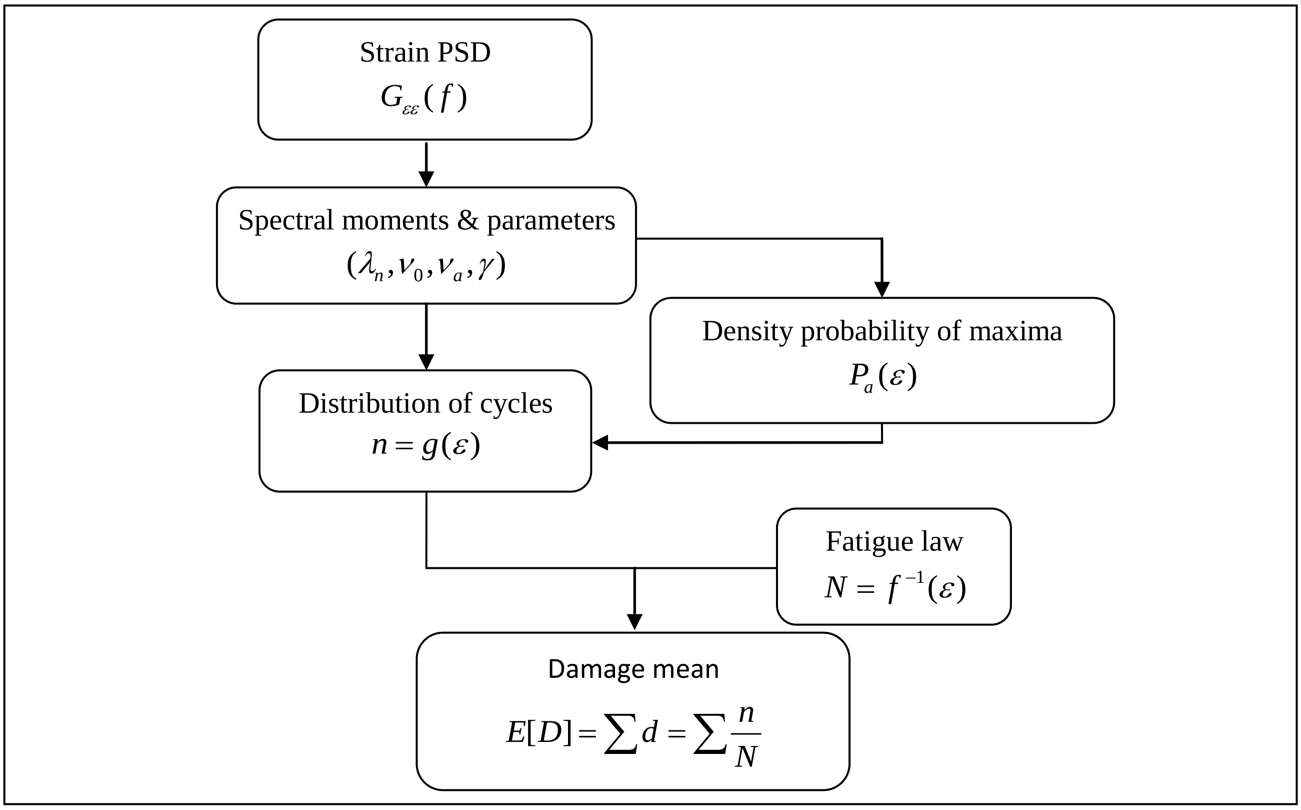


Figure 2: Strain formulation of spectral methods

Neuber correction

This method [10] allows approaching the real state of stress and strain in confined plastic area from an elastic computation. Neuber postulates that the product of stress by strain in a confined plastic area does not depend on the plastic flow (on the vicinity of the yield stress). This allows to establish Eqn. 3:

$$\sigma_e \varepsilon_e = \varepsilon_r \sigma_r \quad (3)$$

To apply the correction we used a cyclic hardening curve (Eqn.4).The approach to determine the real pair of stress and strain is illustrated in Figure 3.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{k'}\right)^{1/n'} \qquad (4)$$

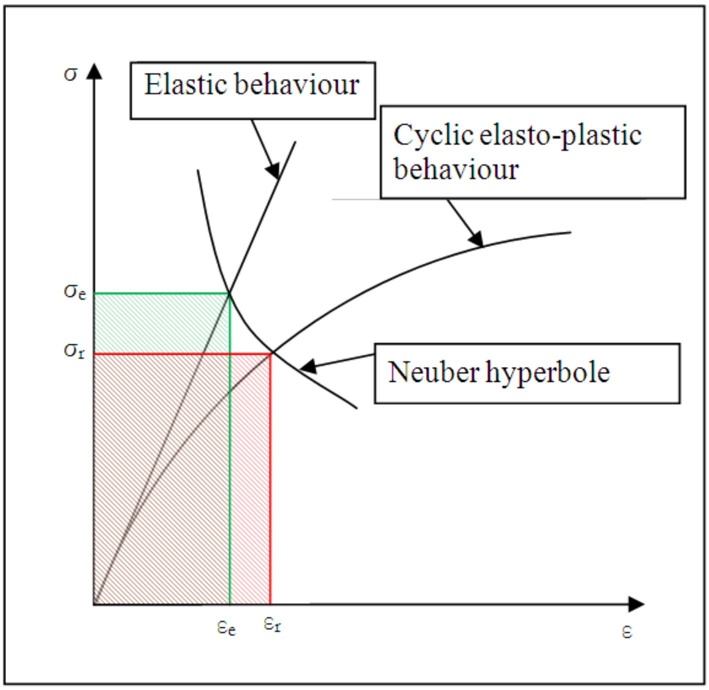


Figure 3: Application of Neuber method

Application of Neuber correction for spectral methods

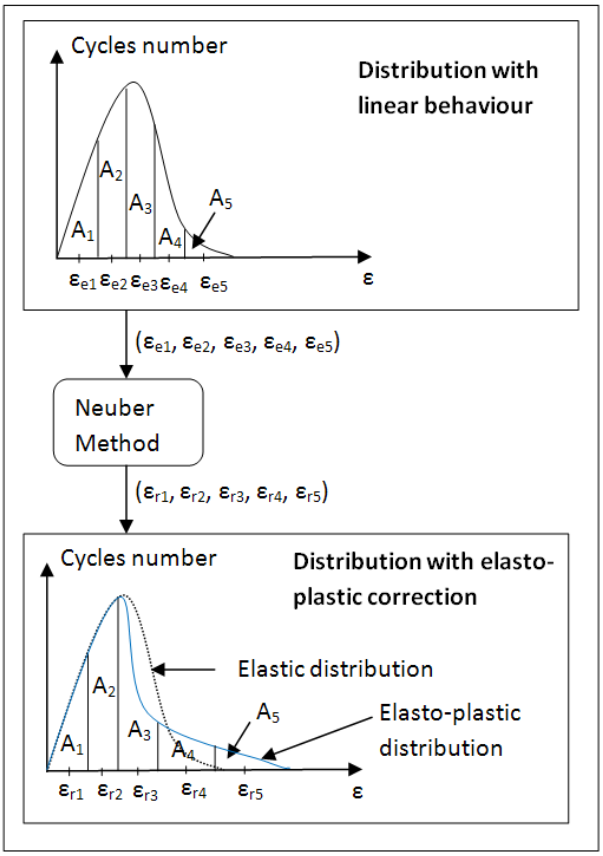


Figure 4: Correction method for the distribution of cycles number in the spectral methods

Spectral methods do not allow to access to loading time history. But they provide for each strain the number of cycles corresponding. To apply the correction, we used Eqn. 3 and Eqn. 4 to our distribution:

$$\left.\begin{aligned} \varepsilon_{ri}\sigma_{ri} &= \frac{\varepsilon_{ei}\varepsilon_{ei}}{E} \\ \varepsilon_{ri} &= \frac{\sigma_{ri}}{E} + \left(\frac{\sigma_{ri}}{k'}\right)^{\frac{1}{n'}} \end{aligned}\right\} \Rightarrow \varepsilon_{ri}$$

We must amuse that the total cycles number from distribution remains unchanged regardless of materials behaviour model. The distribution is defined for a given time interval. In this time interval, the cycle numbers is given by the system frequency and not by the strain or stress amplitudes. We work only on the allocation of the cycles number on the space of reality strain (Fig. 4).This means that the area under the curve remains the same before and after correction, only the cycles added on the high strain modifies the damage.

Results

We propose to validate this method by comparing it to Monte Carlo simulations (MCS). This validation is made from a large panel of PSD profiles.

Monte Carlo Simulations

Monte Carlo simulations are classical means to validate calculation methods in the context of vibration fatigue. It is described in Fig.5 the computation chain MCS applied to our case. In our case, the number of time samples is 1000. This gives a computation time for a sample of 1500s, a computation time by parameter sets of about 2 days.

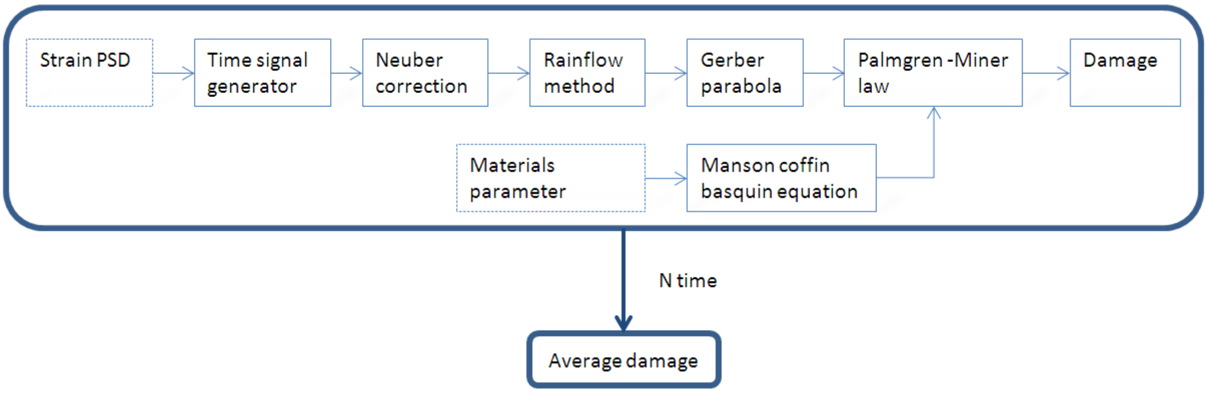


Figure 5 : computation chain of MCS

Input data

Major input data is the PSD strain and fatigue parameters of materials. We use the material parameters of 30NCD8 steel. For the PSD, we selected two template PSD which are realistic mechanical engineering industry. These templates PSD and their parameters are described in Fig.6.

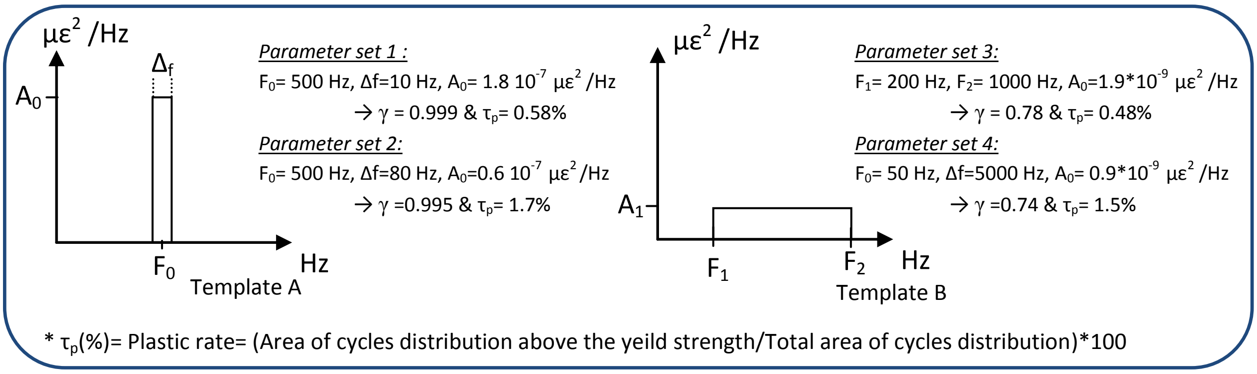


Figure 6: Template PSD strain and their parameter

Some Results

Figure 7 shows the difference in lifetime between the proposed method and the MCS for the template A. We can see also the natural dispersion of the stochastic process for MCS. In Fig.7, the green line represents the average damage obtained by Narrow-Band method in strain formulation and with taking into account the plasticity. The dark line represents the average damage obtained by the conventional Narrow-Band method. The large gap between the green line and the black line is due to taking into account the plasticity in the method described in this article.

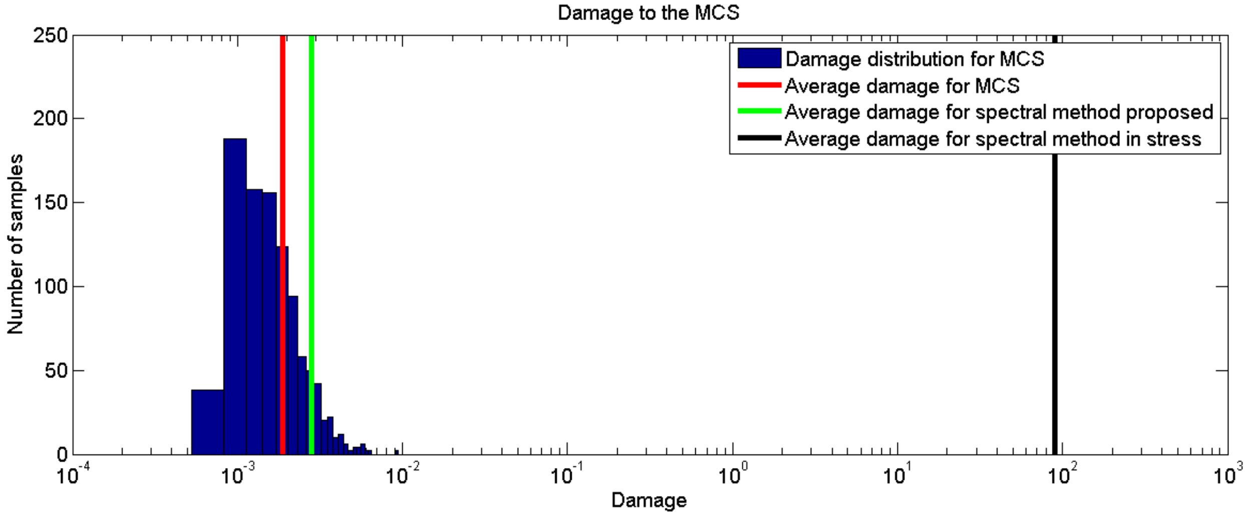


Figure 7: Comparison between the MCS and spectral methods for the template A and parameter set 1

Concluding remarks

-The results show a significant influence in the lifetime estimation in the non-linearity due to plasticity.

-The hypothesis added to taking into account the plasticity is consistent with the results of the MCS.

-These calculations will be repeated on a real case for comparison with experimental results.

References

[1] AFNOR, Fatigue sous sollicitations d'amplitude variable. Méthode Rainflow de comptage, 1993.
[2] Dowing S. & Socie D., Simple rainflow counting algorithms, Int. Journal of Fatigue, 4, 31-40, 1982.
[3] Lemaitre J. & Chaboche J., Mécanique des matériaux solides, Dunod, 1996.
[4] Pitoiset X., Méthodes spectrales pour l'analyse en fatigue des structures métalliques sous chargements aléatoires multiaxiaux, Thèse doctorale de l'université libre de Bruxelle, 2001.
[5] Lalanne C., Vibrations et chocs mécaniques : dommage par fatigue, Vol. Tome 4, Lavoiser, 1999.
[6] Larsen C., Predicting the fatigue life of offshore structures by the single-moment spectral method, Probabilistic Engineering Mechanics, 6, 96-108, 1991.
[7] Benasciutti D. & Tovo R., Spectral methods for lifetime prediction under wide-band stationary random processes, Probabilistic Engineering Mechanics, 27, 867-877, 2006.
[8] Zhao W. & Baker M., On the probability density function of rainflow stress range for stationary Gaussian processes, Int. Journal of Fatigue, 14, 121-135, 1992.
[9] Dirlik T., Application of computers in fatigue analysis, PhD Thesis, Warwick University, 1985.
[10] Neuber H., Theory of stress concentration for shear-strained prismatic bodies with arbitrary nonlinear stress-strain law, Journal Appl. Mech., 28, 544-551, 1961.